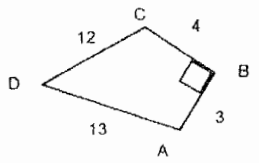
2008 ARML RUNOFF V2
Round I
(45 Minutes) NO CALCULATORS

Print Name _____
School _____ **Grade** _____
e-mail _____

1) Solve for x: $x^3 - x = 2008(x^2 - 1)$.	1
2) Find the sum of the first 25 terms of the following infinite series: $\frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + \ldots + \frac{1}{(2n-1)(2n+1)}$	2
3) If $ax + 3y = 6$ and $4x + by = 9$ represent the same straight line, find the value of $(a + b)$, giving your answer as a fraction in it lowest terms.	3
4) Alfred, Bob, and Calvin compete in the 100 yard dash. In how many different ways can they finish **if ties are allowed?**	4
5) Compute: $\sin 40^\circ(\tan 20^\circ + \cot 20^\circ)$	5
6) Sides $\overline{AB}, \overline{BC}, \overline{CD}$, and $\overline{DA}$ of a convex quadrilateral ABCD have lengths 3, 4, 12, and 13, respectively, and $\angle CBA$ is a right angle. Find the area of the quadrilateral. 	6
7) Given that $\sin A + \sin B = \sqrt{2}$ and $\cos A + \cos B = \sqrt[4]{2}$, what is the smallest value of $\lvert A - B \rvert$, in degrees.	7
8) Solve for x: $x^2(x-3) > 4(x-3)$.	8
9) Two solutions of $x^4 - x^3 - 2x^2 - 4x - 24 = 0$ are pure imaginary numbers. Find these two solutions.	9
10) Solve for y in terms of x: $\frac{1}{2}\log_b x + \log_b y = 0$.	10
11) Solve, for $0 \le x < 2\pi$, the equation: $4\sin x \cos x = \sqrt{3}$.	11
12) Find all solutions (x, y) for the system: $\begin{cases} 4x^2 + 4xy + y^2 = 9 \\ x^2 + 4xy + 4y^2 = 9 \end{cases}$	12

2008 ARML RUNOFF V2
Round II
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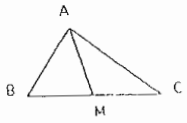
13. If $\frac{1}{a}+\frac{1}{b}=8$, evaluate $3\div\left(\frac{ab}{a+b}\right)$.	13
14. Compute: $\frac{(2008)^3-(2000)^3-8^3}{(2008)(2000)(8)}$	14
15. In the figure shown, in ΔABC, $AB = 4$ and $AC = 8$. If M is the midpoint of $\overline{BC}$ and $AM = 3$, what is the length of $\overline{BC}$? 	15
16. We define a lattice point as a point whose coordinates are integers, zero included. Find the number of lattice points on the boundary and inside the region bounded by the x-axis, the line $x = 4$, and the parabola $y = x^2$.	16
17. In a three-game playoff series, the probability that team A will win each game is twice the probability team B will win. There are no ties. What is the probability that team B will win the series. (The series is over when a team wins 2 games).	17
18. Angles α and β are acute angles such that $\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=1$. If $(\alpha-\beta)=37^\circ$, compute the measure of angle α.	18
19. Point D is on $\overline{BC}$ of ΔABC. If $m\angle CAD = m\angle DAB = 60^\circ$, $AC = 3$ and $AB = 6$, find the length of $\overline{AD}$ (Trig works; geometry works.)	19
20. Any two spheres are similar solids. The volumes are 288π and 972π, respectively. Find the ratio of their areas.	20
21. Solve for x: $x=\sqrt{1+\frac{1}{x}}-1$.	21
22. If x is a real number, when is the quantity $(1+\lvert x\rvert)(1+x)$ positive?	22
23. Find the equation of the line passing through (2, -1) and perpendicular to the line through (4, 3) and (-2, 5).	23
24. Compute the length of the tangent segment from the origin in the circle that passes through the points (3, 4), (6, 8), and (5, 13).	24